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Bidirectional Full-Duplex MIMO Links at Large-System Limit

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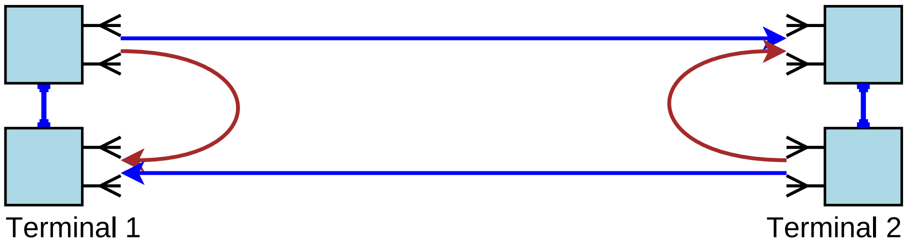
Introduction

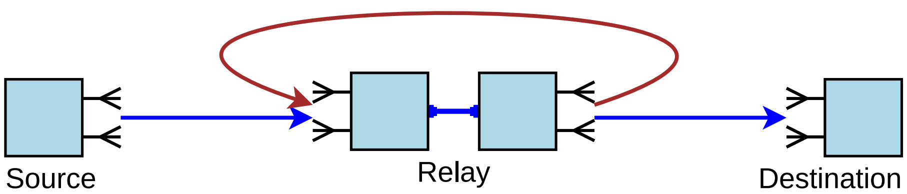
General Topic: Full-Duplex Wireless

- *"Full-duplex"* wireless communication
  - = systems where some node(s) may transmit and receive simultaneously on a *single* frequency band
- Progressive physical/link-layer *frequency-reuse* concept
  - = up to double spectral efficiency at system level, if the significant technical problem of *self-interference* is tackled
- Transmission and reception should use the band for the same amount of time to make the most of full duplex
  - ▷ (a)symmetry of traffic pattern, i.e., *requested* rates in the two simultaneous directions
  - ▷ (a)symmetry of channel quality, i.e., *achieved* rates in the two simultaneous directions

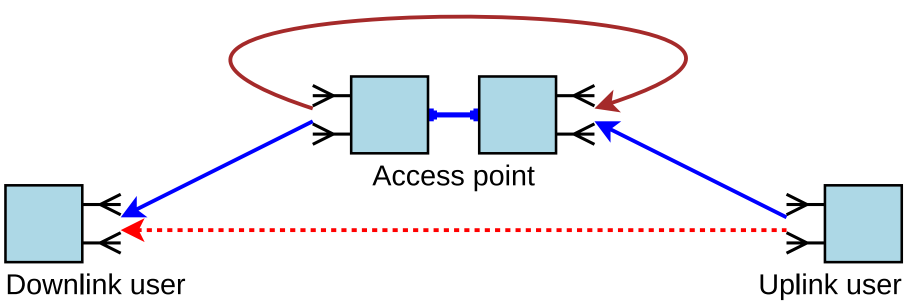
Full-Duplex Communication Scenarios



1) Bidirectional communication link between two terminals
   - Asymmetric traffic (typically)
   - Symmetric channels (roughly)

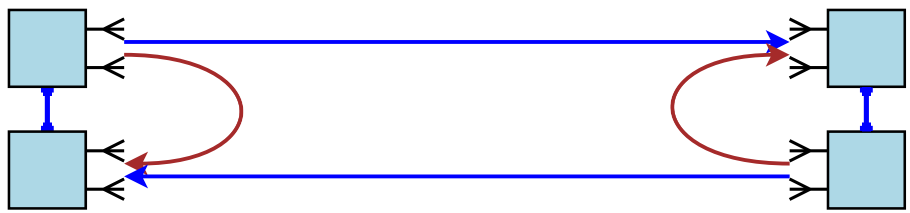


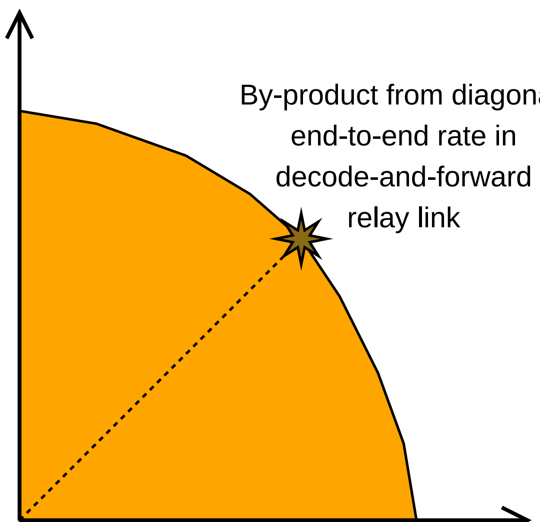
2) Multihop relay link
   - Symmetric traffic
   - Asymmetric channels
   - Direct link may be useful



3) Simultaneous down- and uplink for two half-duplex users
   - Asymmetric traffic
   - Asymmetric channels
   - Inter-user interference!

Scope: Rate Regions in Two-Way Communication

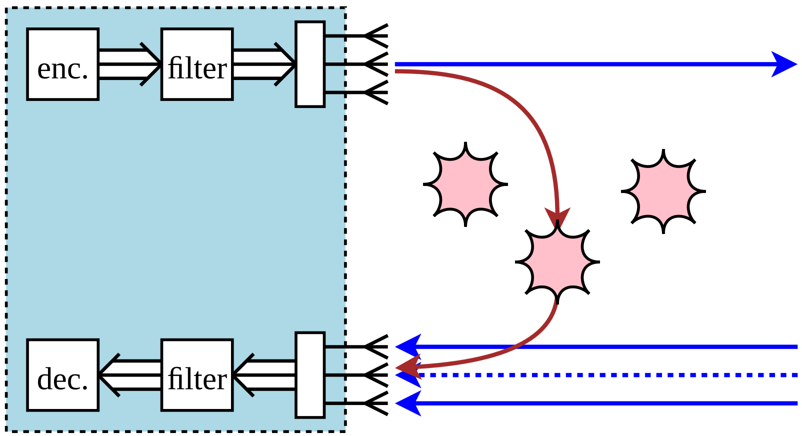




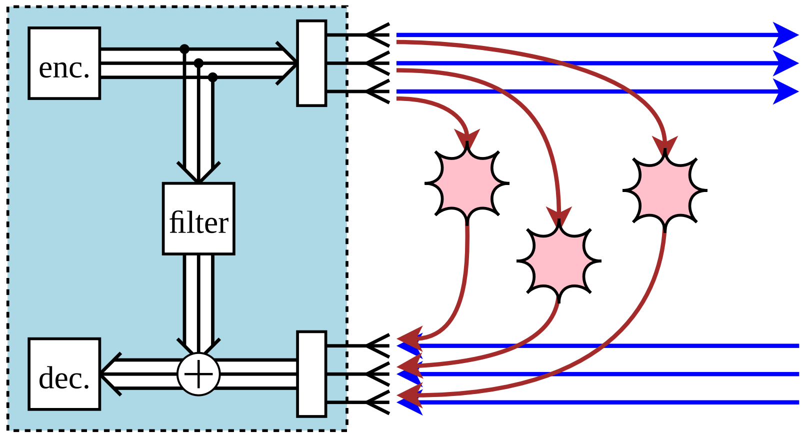
- Bidirectional full-duplex multiantenna (MIMO) link
  - ▷ at the large-system limit
  - ▷ with asymmetric traffic
  - ▷ assuming symmetric setup for numerical results
- Achievable rate regions by controlling
  - ▷ spatial multiplexing
  - ▷ time sharing
- The analysis is based on the *replica method* borrowed from statistical physics

Focus: Suppression vs. Cancellation without Tx Noise

Spatial-domain suppression:



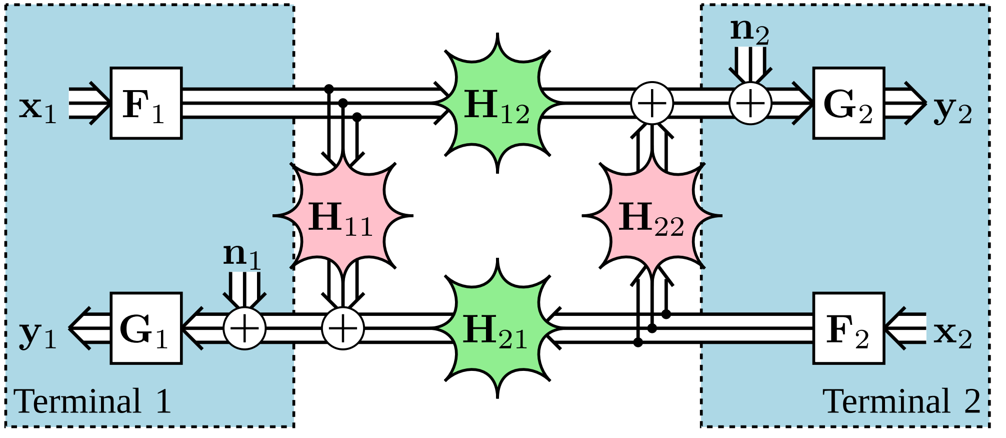
Time-domain cancellation:



- The link needs efficient self-interference mitigation at both ends
  - ▷ **Suppression**: forming eigenbeams to transmit and receive in orthogonal directions (“null-space projection”)
  - ▷ **Cancellation**: subtracting the interfering signal before decoder
- Both schemes can eliminate interference, but suppression is possible only at the cost of consuming spatial degrees of freedom

System Model without Tx Noise

Signal Model



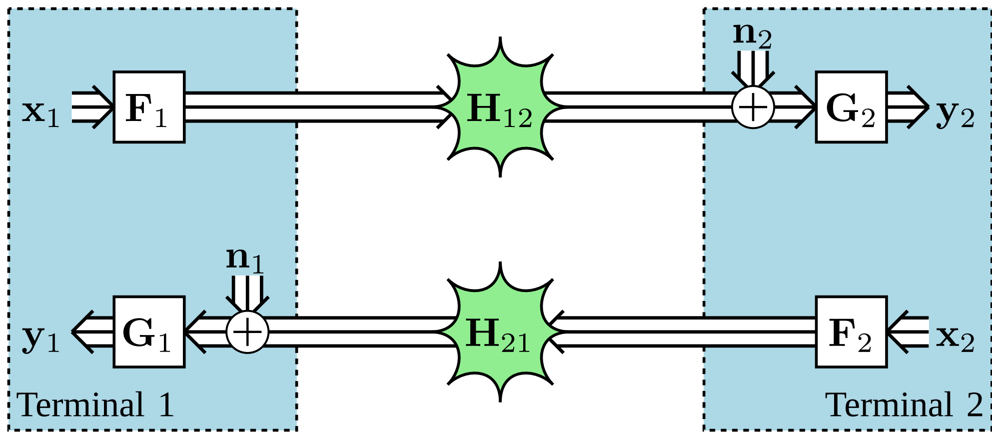
- Terminal $i \in \{1, 2\}$ has M_i transmit and N_i receive antennas
- In communication direction $ij \in \{12, 21\}$:

$$\mathbf{y}_j = \mathbf{G}_j \mathbf{H}_{ij} \mathbf{F}_i \mathbf{x}_i + \mathbf{G}_j \mathbf{H}_{jj} \mathbf{F}_j \mathbf{x}_j + \mathbf{G}_j \mathbf{n}_j$$

  - ▷ The link reserves $\hat{M}_i$ transmit and $\hat{N}_j$ receive streams for spatial multiplexing after self-interference mitigation

- Terminal i does not know $\mathbf{H}_{ij}$ but Terminal j knows $\mathbf{H}_{ij}$ and $\mathbf{H}_{jj}$

Spatial-Domain Suppression

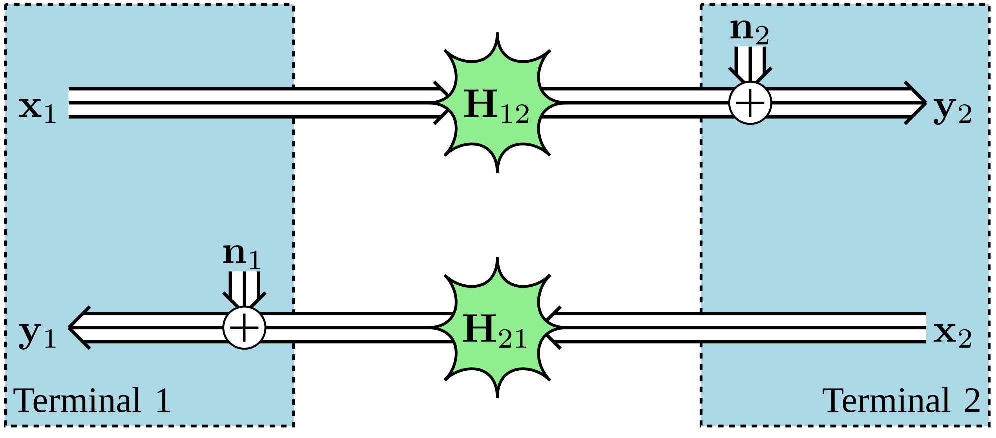


- Suppression exploits the transmit and receive beamforming filters:

$$\mathbf{F}_j \in \mathbb{C}^{M_j \times \hat{M}_j} \quad \text{and} \quad \mathbf{G}_j \in \mathbb{C}^{\hat{N}_j \times N_j}$$

  - ▷ Orthonormal spatial streams: $\mathbf{F}_j^H \mathbf{F}_j = \mathbf{I}$ and $\mathbf{G}_j \mathbf{G}_j^H = \mathbf{I}$
    - – Maximum for full-rank $\mathbf{H}_{jj}$ is $\hat{M}_j + \hat{N}_j = \max\{M_j, N_j\}$
- Self-interference is eliminated in Terminal j if $\boxed{\mathbf{G}_j \mathbf{H}_{jj} \mathbf{F}_j = \mathbf{0}}$
  - ▷ Implemented using the SVD of $\mathbf{H}_{jj}$ (for instance)

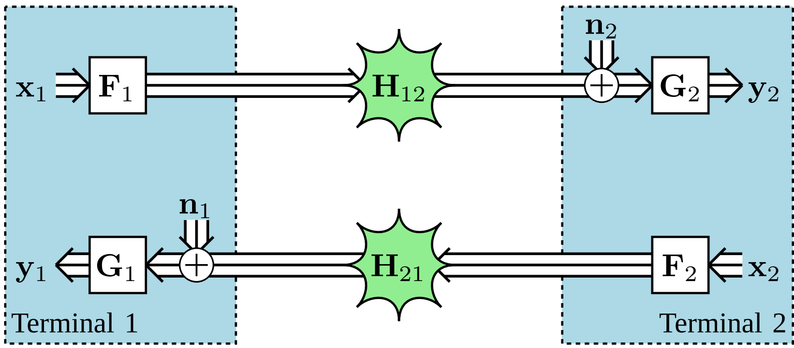
Time-Domain Cancellation



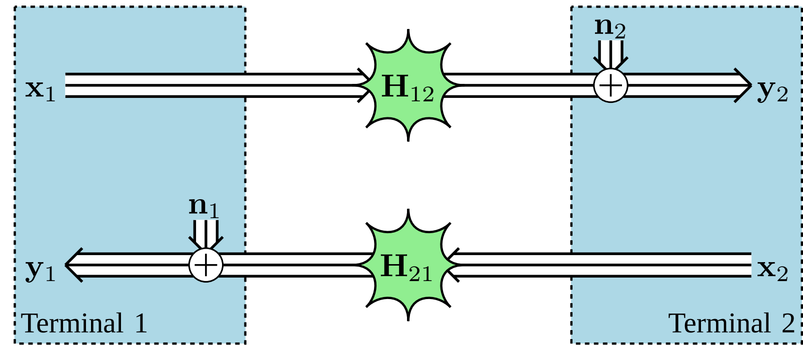
- Cancellation is based on the subtraction of the interfering signal so that decoder input becomes $\boxed{\mathbf{y}_j - \mathbf{G}_j \mathbf{H}_{jj} \mathbf{F}_j \mathbf{x}_j}$
  - Terminal $j \in \{1, 2\}$ needs to know its own transmitted signal $\mathbf{x}_j$ which is not required with spatial-domain suppression
- All spatial degrees of freedom can be reserved for multiplexing
  - $\hat{M}_j = M_j$, $\hat{N}_j = N_j$ and $\mathbf{F}_j = \mathbf{I}$, $\mathbf{G}_j = \mathbf{I}$ in the analysis

Spatial-Division Multiplexing

Spatial-domain *suppression*:



Time-domain *cancellation*:



- After mitigation, the signal model is transformed to

$$\mathbf{y}_j = \mathbf{G}_j \mathbf{H}_{ij} \mathbf{F}_i \mathbf{x}_i + \mathbf{G}_j \mathbf{n}_j, \qquad \mathcal{E}\{\mathbf{x}_i \mathbf{x}_i^H\} = (1/\hat{M}_i)\,\mathbf{I}$$

  - Transmitter side: standard open-loop spatial multiplexing of independent Gaussian streams into $\mathbf{x}_i$
  - Receiver side: joint decoding for $\mathbf{y}_j$ knowing $\mathbf{G}_j \mathbf{H}_{ij} \mathbf{F}_i \in \mathbb{C}^{\hat{N}_j \times \hat{M}_i}$
- Time sharing between different stream configurations in order to make the achievable rate region convex with suppression

Analytical Results

Mutual Information

- We are interested in evaluating the average transmission rate as

$$\boxed{R_{ij} = \mathcal{E}\{\log\det(\mathbf{I} + \frac{1}{\hat{M}_i}\mathbf{G}_j\mathbf{H}_{ij}\mathbf{F}_i(\mathbf{G}_j\mathbf{H}_{ij}\mathbf{F}_i)^H)\}}$$

  over the joint distribution of random matrices $\mathbf{G}_j$, $\mathbf{H}_{ij}$, and $\mathbf{F}_i$
- Instead, we begin from the definition of mutual information:

$$\frac{R_{ij}}{\hat{M}_i} = \frac{\mathcal{E}\{\log p(\mathbf{y}_j \mid \mathbf{x}_i, \mathbf{G}_j, \mathbf{H}_{ij}, \mathbf{F}_i)\}}{\hat{M}_i} - \frac{\mathcal{E}\{\log \mathsf{E}_{\mathbf{x}_i}\{p(\mathbf{y}_j \mid \mathbf{x}_i, \mathbf{G}_j, \mathbf{H}_{ij}, \mathbf{F}_i)\}\}}{\hat{M}_i}$$

  where $p(\cdot \mid \cdot)$ is the Gaussian posterior probability
- The above expression can be transformed to

$$\frac{R_{ij}}{\hat{M}_i} = -\frac{\hat{N}_j}{\hat{M}_i} - \frac{1}{\hat{M}_i}\lim_{u\to 0}\frac{\partial}{\partial u}\log\mathcal{E}\{\mathsf{E}_{\mathbf{x}_i}\{\exp(-\|\mathbf{y}_j - \mathbf{G}_j\mathbf{H}_{ij}\mathbf{F}_i\mathbf{x}_i\|^2)\}^u\}$$

  where the first term is trivial and the second term comes from the identity $\lim_{u\to 0}\frac{\partial}{\partial u}\log\mathcal{E}\{Z^u\} = \mathcal{E}\{\log Z\}$

Replica Method and Integration

- With $\Delta\mathbf{x}_a = \mathbf{x}_0 - \mathbf{x}_a$, the replica trick amounts to evaluating

$$\frac{R_{ij}}{\hat{M}_i} = -\frac{\hat{N}_j}{\hat{M}_i} - \lim_{\hat{M}_i\to\infty}\frac{1}{\hat{M}_i}\lim_{u\to 0}\frac{\partial}{\partial u}\log\mathcal{E}\{\prod_{a=1}^{u}\mathrm{e}^{-\|\hat{M}_i^{-1/2}\mathbf{G}_j\mathbf{H}_{ij}\mathbf{F}_i\Delta\mathbf{x}_a+\mathbf{G}_j\mathbf{n}_j\|^2}\}$$

  where u is an integer inside $\log$ but a real number outside $\log$ (!?)
- After Gaussian integration over $\mathbf{n}_j$ and $\mathbf{v}_a = \hat{M}_i^{-1/2}\mathbf{H}_{ij}\mathbf{F}_i\Delta\mathbf{x}_a$,

$$\frac{R_{ij}}{\hat{M}_i} = -\frac{\hat{N}_j}{\hat{M}_i} - \lim_{\hat{M}_i\to\infty}\frac{1}{\hat{M}_i}\lim_{u\to 0}\frac{\partial}{\partial u}\log\mathcal{E}\{\mathrm{e}^{G(\mathbf{Q},\mathbf{D}_j)}\}$$

  where $\{\mathbf{Q}\}_{a,b} = \frac{1}{\hat{M}_i}\mathbf{x}_b^H\mathbf{x}_a$ and $\mathbf{D}_j = \mathbf{T}_j^T\mathbf{T}_j$ is binary and diagonal
- If the limits can be swapped, the saddle-point method implies

$$\frac{R_{ij}}{\hat{M}_i} = -\frac{\hat{N}_j}{\hat{M}_i} - \lim_{u\to 0}\frac{\partial}{\partial u}\lim_{\hat{M}_i\to\infty}\frac{1}{\hat{M}_i}\log\mathsf{E}_{\mathbf{D}_j}\{\exp(\hat{M}_i\operatorname*{extr}_{\mathbf{Q},\tilde{\mathbf{Q}}}T(\mathbf{Q},\tilde{\mathbf{Q}},\mathbf{D}_j))\}$$

  where $T(\mathbf{Q},\tilde{\mathbf{Q}},\mathbf{D}_j) = \frac{1}{\hat{M}_i}G(\mathbf{Q},\mathbf{D}_j) - \mathrm{tr}(\mathbf{Q}\tilde{\mathbf{Q}}) + \log M(\tilde{\mathbf{Q}})$

Replica Symmetry Assumption and Extremization

- Before extremization, $T(\mathbf{Q}, \tilde{\mathbf{Q}}, \mathbf{D}_j)$ is transformed by replica symmetry
  ($\mathbf{Q} = \mathbf{I}_{u+1}(p-q) + \mathbf{1}_{(u+1)\times(u+1)}q$ and $\tilde{\mathbf{Q}} = \mathbf{I}_{u+1}(\tilde{p}-\tilde{q}) + \mathbf{1}_{(u+1)\times(u+1)}\tilde{q}$)
  to $T_u(p, q, \tilde{p}, \tilde{q}) = -u\frac{\hat{N}_j}{\hat{M}_i}\log(1+\bar{\gamma}_{ij}(p-q)) - (u+1)(p\tilde{p}+uq\tilde{q}) + \log M(\tilde{\mathbf{Q}})$
- Matrix $\mathbf{D}_j$ also disappears and we get a tractable form as

$$\frac{R_{ij}}{\hat{M}_i} = -\lim_{u\to 0}\frac{\partial}{\partial u}\operatorname*{extr}_{p,q,\tilde{p},\tilde{q}} T_u(p, q, \tilde{p}, \tilde{q})$$

  which matches to the case of an i.i.d. Gaussian $\hat{M}_i \times \hat{N}_j$ channel
- Finally, we may exploit existing proofs (e.g., by Verdú) to obtain

$$\boxed{\frac{R_{ij}}{\hat{M}_i} \simeq \log\left(1+\frac{\hat{N}_j}{\hat{M}_i}\cdot\frac{\bar{\gamma}_{ij}}{1+E}\right) + \frac{\hat{N}_j}{\hat{M}_i}\left(\log(1+E) - \frac{E}{1+E}\right)}$$

  where $E = \bar{\gamma}_{ij}(p-q)$ is a solution to $\frac{\bar{\gamma}_{ij}}{E} = 1 + \frac{\hat{N}_j}{\hat{M}_i}\cdot\frac{\bar{\gamma}_{ij}}{1+E}$
  - ▷ The achievable transmission rates of the two directions are indirectly coupled via $\hat{M}_j + \hat{N}_j = \max\{M_j, N_j\}$

Numerical Results

Example Setups

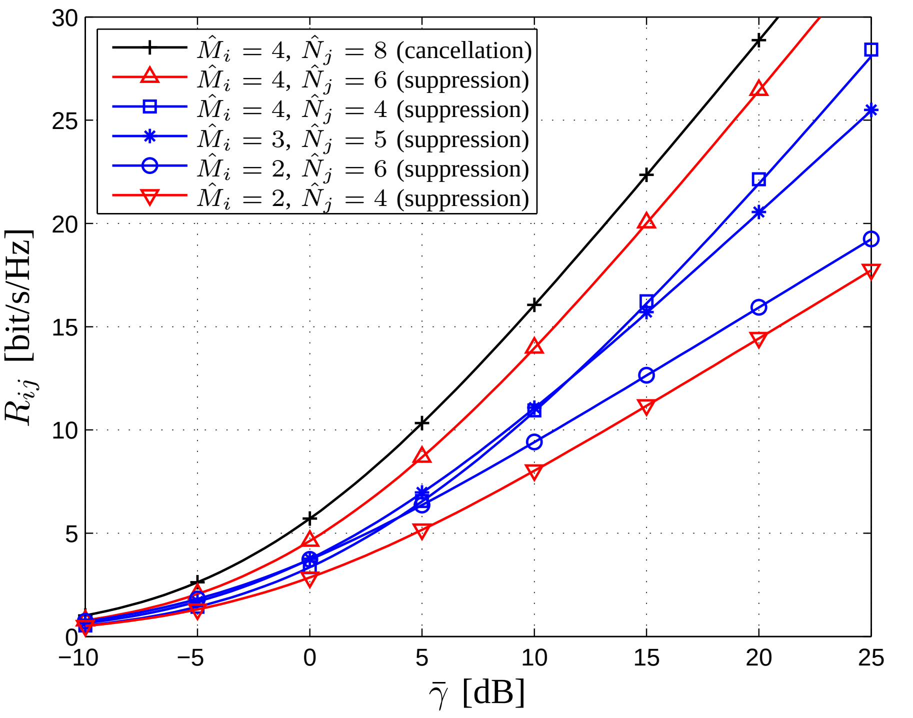
- The numerical results concentrate on symmetric systems where
  - ▷ $M = M_1 = M_2$
  - ▷ $N = N_1 = N_2$
  - ▷ $\bar{\gamma} = \bar{\gamma}_{12} = \bar{\gamma}_{21}$
- However, some asymmetry should be taken into account
  - ▷ Requested rates may be different in the two directions, reflecting typical downlink/uplink imbalance (R_{12}/R_{21})
  - ▷ There may be transmit/receive antenna imbalance (M/N)
    - – At the large-system limit, M and N grow asymptotically
- In summary, there are three key parameters to explore:

$$\boxed{R_{12}/R_{21}} \qquad \boxed{\bar{\gamma}} \qquad \boxed{M/N}$$

Transmission Rate vs. SNR

- When
  - $M = 4$
  - $N = 8$

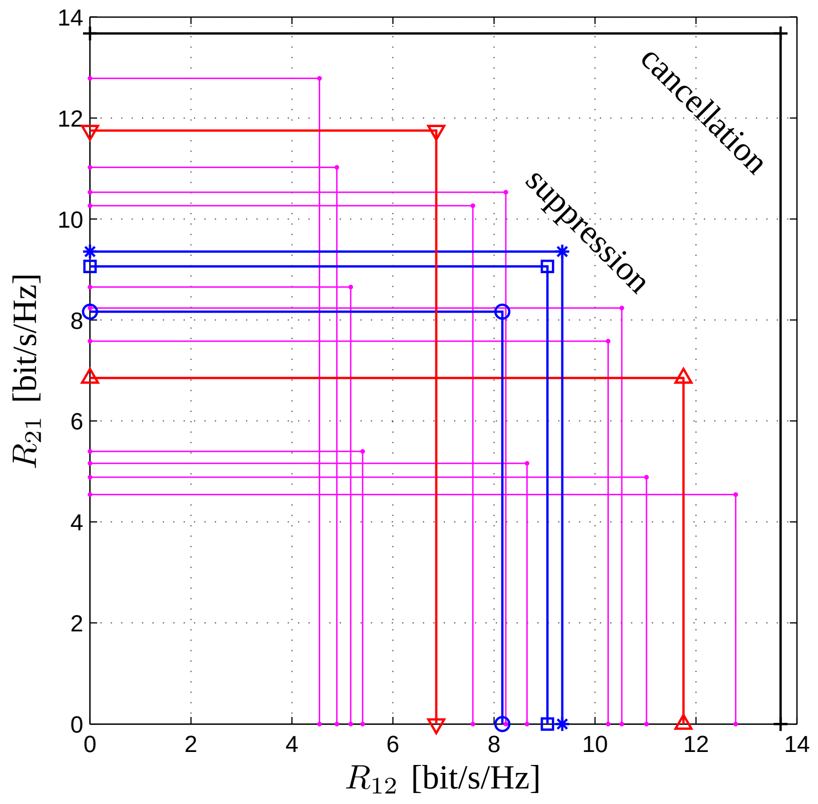
  a) lines:
  asymptotic *analytical* values projected to this finite case
  b) markers:
  accurate *simulated* values



- The asymptotic results are useful also for not-so-large systems
- Trade-off (indirect coupling) between rates in two directions:
  When choosing $(\hat{M}_i, \hat{N}_j)$ as a stream configuration in one direction, the opposite configuration becomes $(\hat{M}_j, \hat{N}_i) = (8 - \hat{N}_j, 8 - \hat{M}_i)$

Achievable Rate Regions (1)

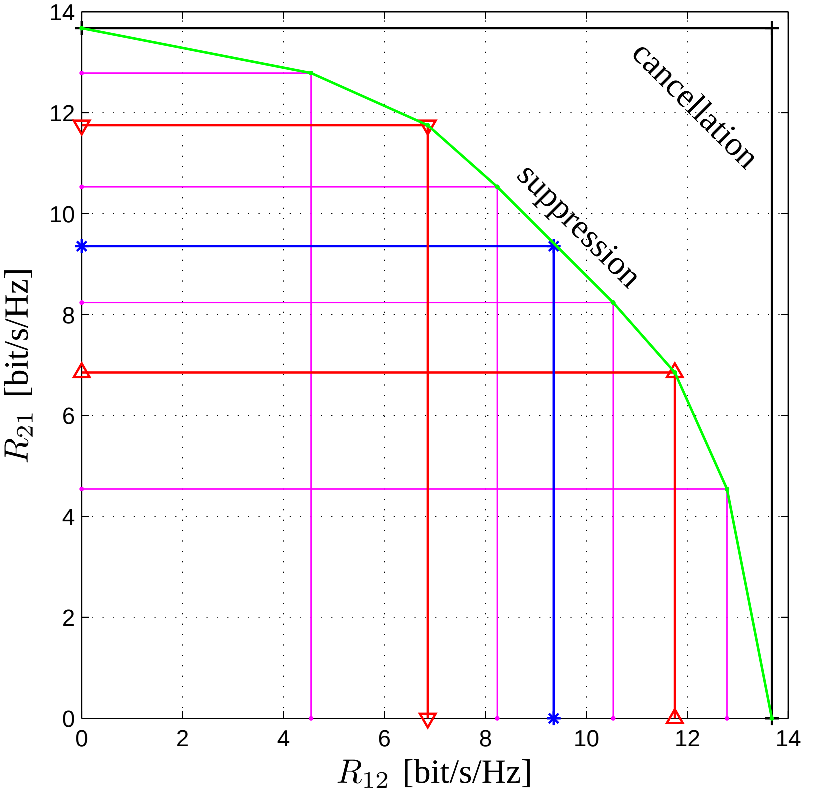
- When
  - $M = 4$
  - $N = 8$
  - $\bar{\gamma} = 8\text{dB}$
- Varying $\hat{M}_1$ and $\hat{M}_2$ which sets $\hat{N}_1 = 8 - \hat{M}_1$ $\hat{N}_2 = 8 - \hat{M}_2$ for suppression



- Each stream configuration $(\hat{M}_1, \hat{M}_2)$ renders a rectangular region
  - Suppression: 16 different two-way regions and two degenerate cases where data is transmitted in one direction only

Achievable Rate Regions (2)

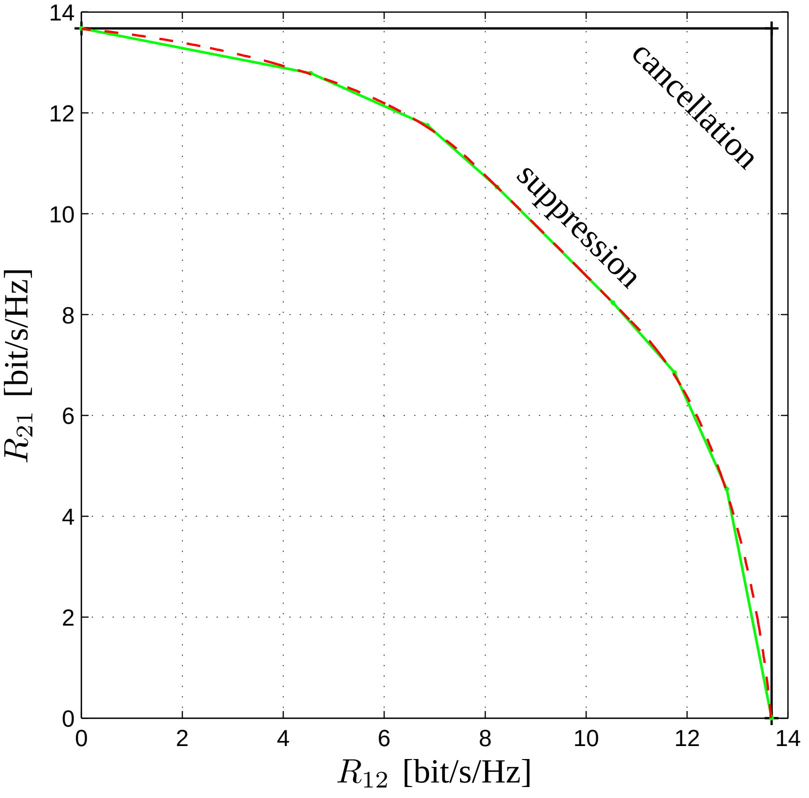
- When
  - $M = 4$
  - $N = 8$
  - $\bar{\gamma} = 8\text{dB}$
- Varying $\hat{M}_1$ and $\hat{M}_2$ which sets $\hat{N}_1 = 8 - \hat{M}_1$ $\hat{N}_2 = 8 - \hat{M}_2$ for suppression



- The complete rate region is achieved by *time sharing* between different fixed stream configurations $(\hat{M}_1, \hat{M}_2)$
  - The convex hull of the union of rectangular rate regions

Achievable Rate Regions (3)

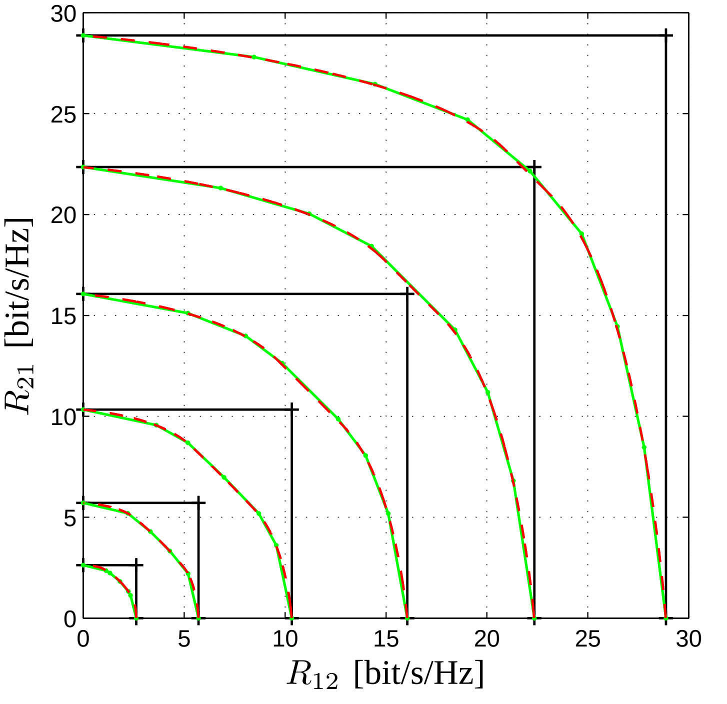
- When
  - $M = 4$
  - $N = 8$
  - $\bar{\gamma} = 8\text{dB}$
- Varying continuously $\hat{M}_1/M$ and $\hat{M}_2/M$
- Using time sharing when $R_{12} \sim R_{21}$



- Rate region projected from the asymptotic analytical results (dashed line) matches well with the finite-case simulations

Achievable Rate Regions vs. SNR

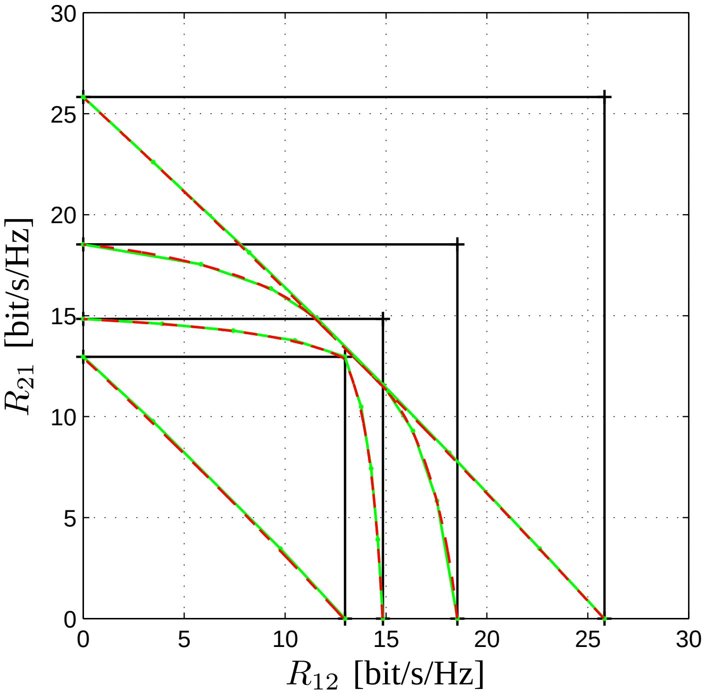
- When
  - $(M, N) = (4, 8)$
  - $\bar{\gamma} = 20\text{dB}$
    $= 15\text{dB}$
    $= 10\text{dB}$
    $= 5\text{dB}$
    $= 0\text{dB}$
    $= -5\text{dB}$



- The absolute rates increase with the SNR value, as expected, but otherwise it affects only slightly the shape of rate regions
- Asymmetric traffic can be supported without time sharing

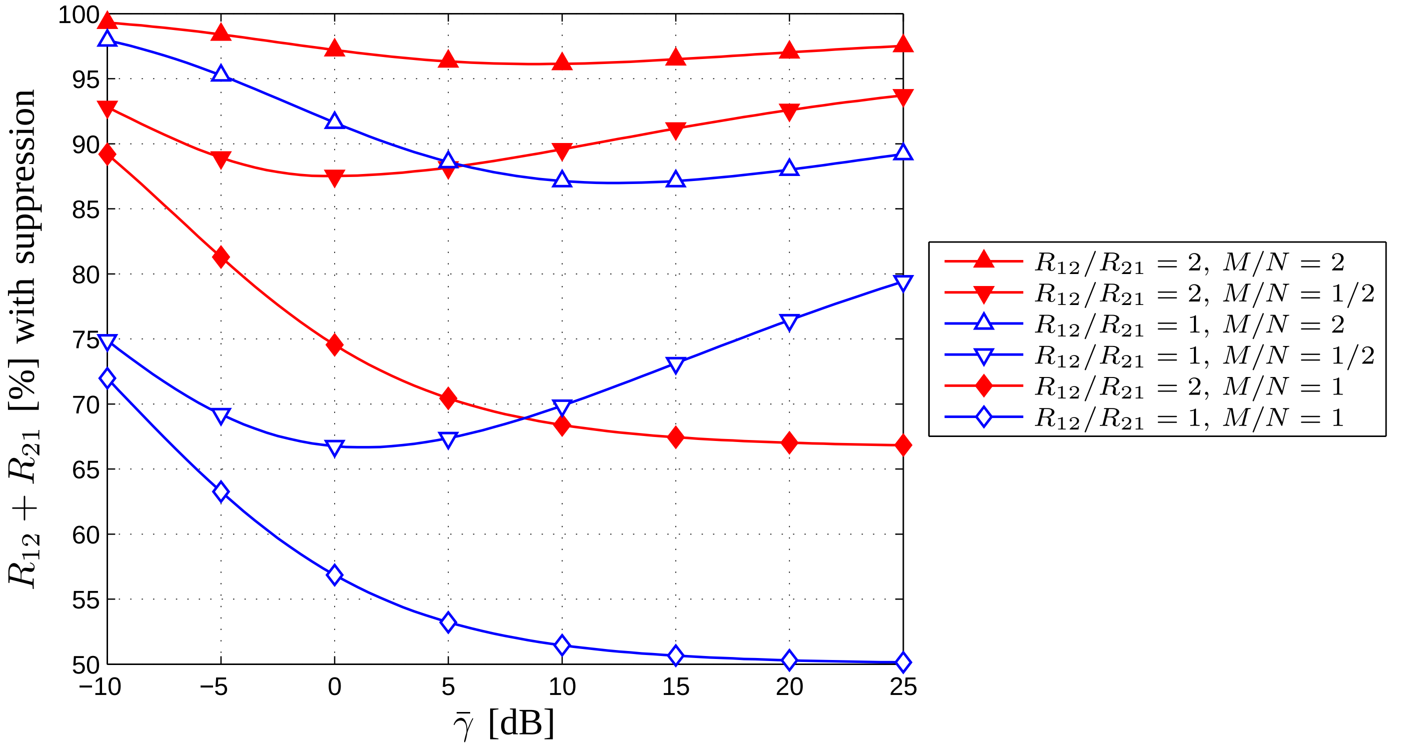
Achievable Rate Regions vs. Antenna Imbalance

- When
  - $(M, N) = (8, 8)$
    $= (4, 8)$
    $= (8, 4)$
    $= (4, 4)$
  - $\bar{\gamma} = \ 12\text{dB}$



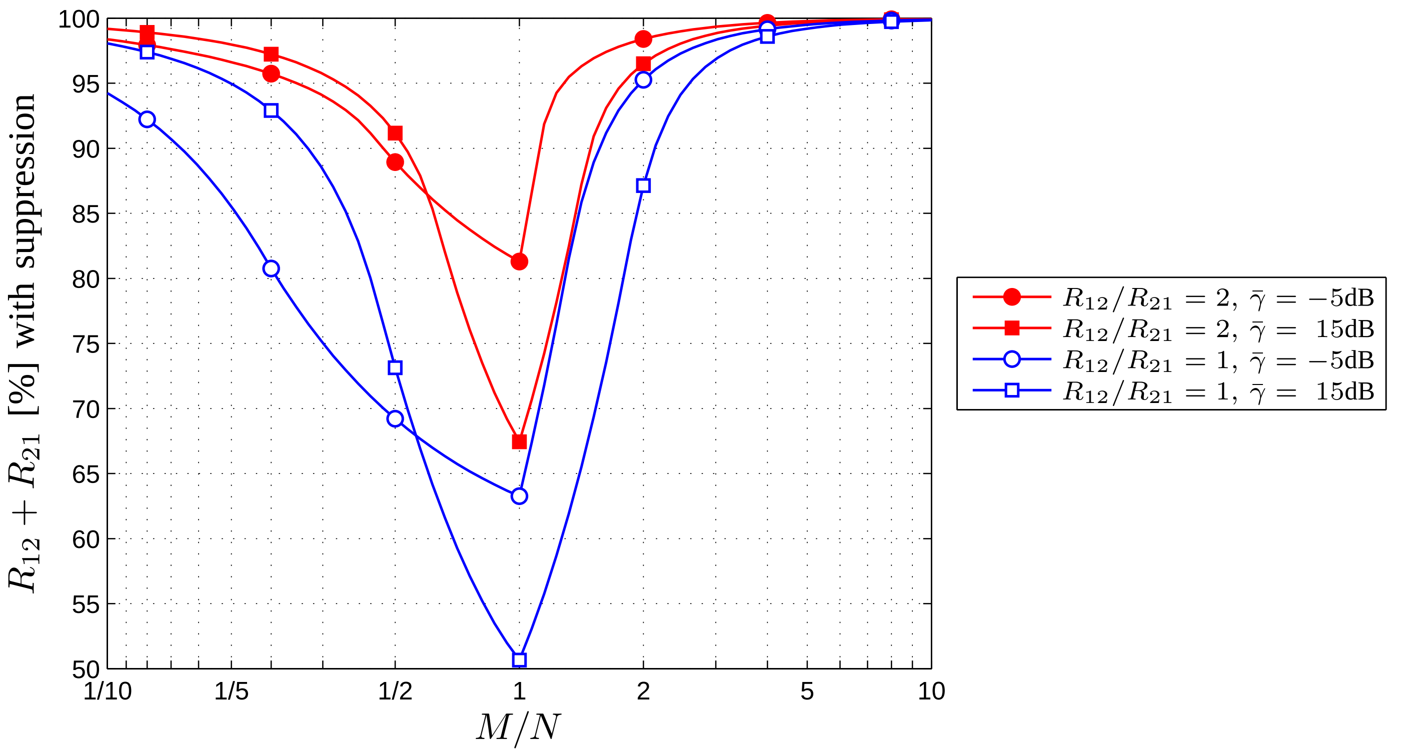
- Transmit/receive antenna imbalance (M/N) affects significantly the shape of the rate regions with spatial-domain suppression
- The rate region of suppression is always inside that of cancellation

Suppression vs. Cancellation (SNR)



- SNR defines whether the performance is limited by transmit-side multiplexing gain or receive-side array gain
- Worst case: equal number of transmit and receive antennas

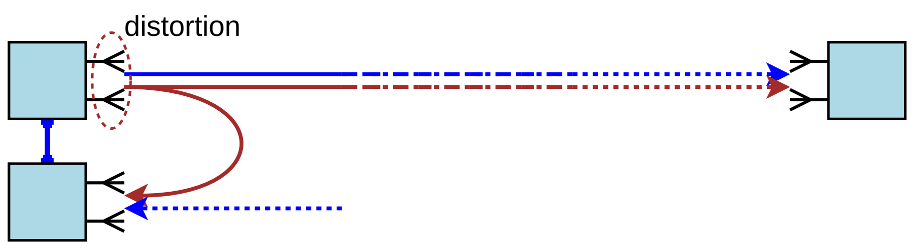
Suppression vs. Cancellation (Antenna Imbalance)



- Transmit/receive antenna imbalance is a critical factor when characterizing the rate loss of suppression versus cancellation
- Having more transmit antennas than receive antennas is preferred

Transmitter Noise and M(ism)atched Decoding

Transmitter Noise and M(ism)atched Decoding

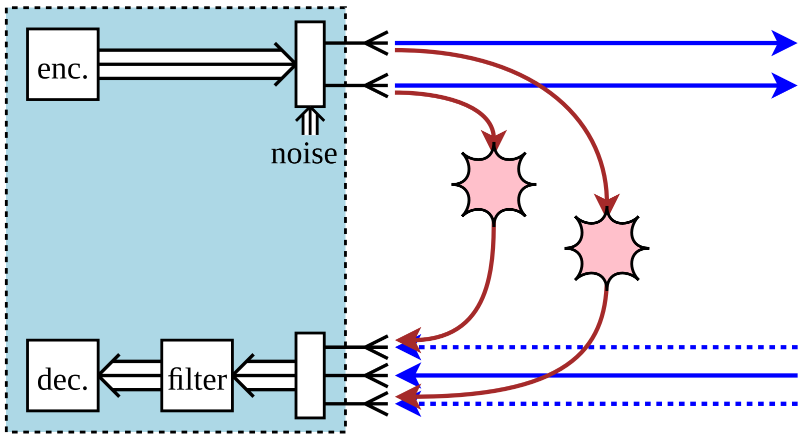


- *Unknown* transmit-side noise due to analog imperfections
  - ▷ nonlinear distortion, e.g., power amplifier (PA)
  - ▷ measured with EVM

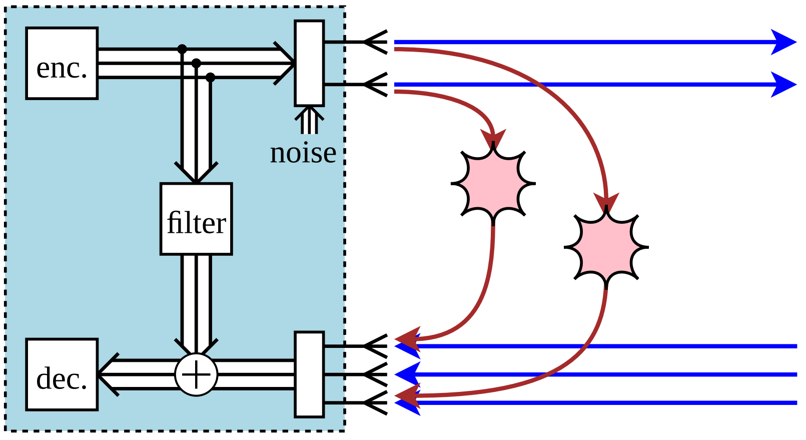
- *Feedback* transmit-side noise may be on a par with the far-end signal due to the high gain of the near-end interference channel
  - ▷ *Feedforward* transmit-side noise can be neglected since it is typically below receive-side noise after channel attenuation
- Mitigation transparently around the actual multiplexing protocol which can operate without being aware of self-interference
  - ▷ *Mismatched* detection and decoding due to unexpected noise

Focus: Self-interference Mitigation with Tx Noise

Spatial-domain suppression:



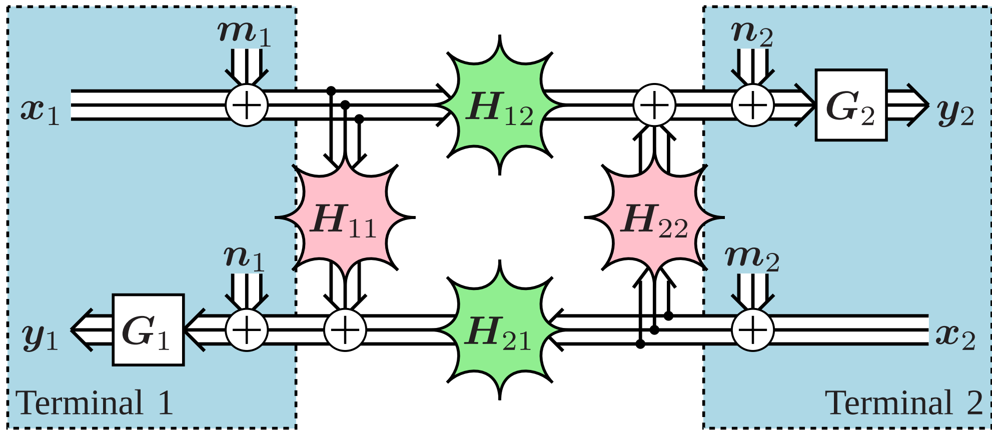
Time-domain cancellation:



- The link needs efficient self-interference mitigation at both ends
  - ▷ **Suppression**: receiving only in the null space of interference
  - ▷ **Cancellation**: subtracting the interfering signal before decoder
- Both can eliminate the data-dependent part of self-interference
- Suppression eliminates also the self-induced transmit-side noise, at the cost of consuming some spatial degrees of freedom

System Model with Tx Noise

Signal Model

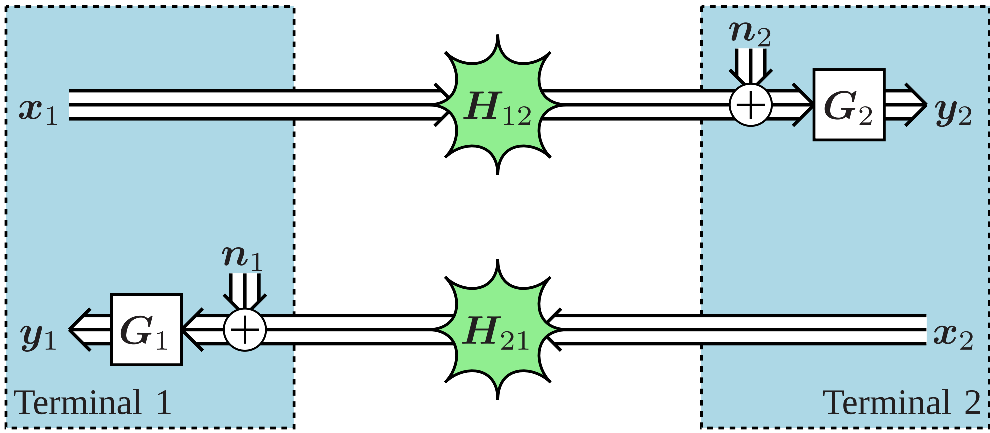


- Terminal $i \in \{1,2\}$ has M_i transmit and N_i receive antennas
- In communication direction $ij \in \{12, 21\}$:

$$\mathbf{y}_j = \mathbf{G}_j \mathbf{H}_{ij}(\mathbf{x}_i + \boldsymbol{m}_i) + \mathbf{G}_j \mathbf{H}_{jj}(\mathbf{x}_j + \boldsymbol{m}_j) + \mathbf{G}_j \mathbf{n}_j$$

  - ▷ noise terms $\boldsymbol{m}_i$ and $\boldsymbol{m}_j$ due to transmitter imperfections
  - ▷ $\hat{N}_j$ receive streams remain after self-interference mitigation
- Terminal i does not know $\mathbf{H}_{ij}$ but Terminal j knows $\mathbf{H}_{ij}$ and $\mathbf{H}_{jj}$

Spatial-Domain Suppression

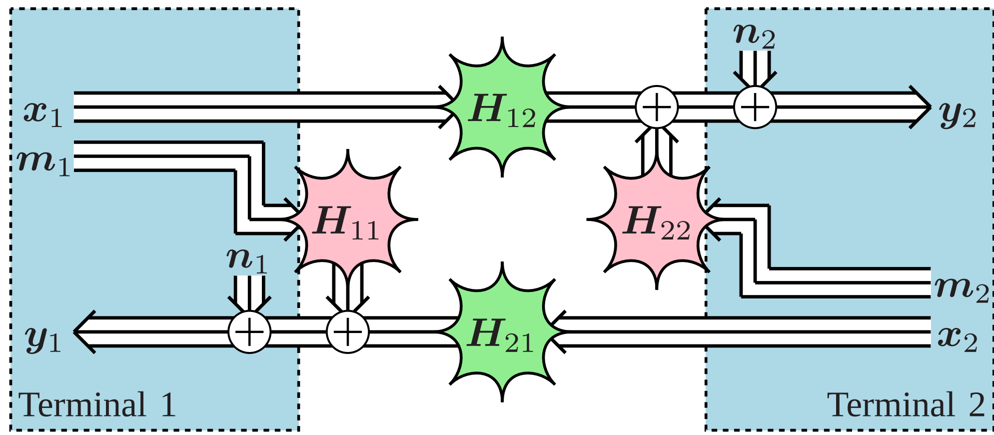


- In Terminal $j \in \{1, 2\}$ after suppression using $\mathbf{G}_j$ of rank $\hat{N}_j$:

$$\mathbf{y}_j = \mathbf{G}_j(\mathbf{H}_{ij}\mathbf{x}_i + \underbrace{\mathbf{H}_{ij}\boldsymbol{m}_i}_{\approx \mathbf{0}}) + \underbrace{\mathbf{G}_j\mathbf{H}_{jj}(\mathbf{x}_j + \boldsymbol{m}_j)}_{\text{eliminated when } \mathbf{G}_j\mathbf{H}_{jj}=\mathbf{0}} + \mathbf{G}_j\mathbf{n}_j$$

- $\hat{N}_j = N_j - M_j$ if $\mathbf{H}_{jj}$ has full rank, thus requiring $N_j > M_j$
- When enclosing any conventional (e.g., half-duplex) transceiver by transparent suppression, it still performs *matched decoding*

Time-Domain Cancellation



- In Terminal $j \in \{1, 2\}$ after cancellation presuming $\mathbf{G}_j = \mathbf{I}$:

$$\mathbf{y}_j = \mathbf{H}_{ij}\mathbf{x}_i + \underbrace{\mathbf{H}_{ij}\boldsymbol{m}_i}_{\approx \mathbf{0}} + \underbrace{\mathbf{H}_{jj}\mathbf{x}_j}_{\text{eliminated}} + \mathbf{H}_{jj}\underbrace{\boldsymbol{m}_j}_{\text{unknown!}} + \mathbf{n}_j$$

- $\hat{N}_j = N_j$, i.e., all degrees of freedom are saved for data reception
- Conventional receivers may adapt imperfectly to the presence of unexpected transmitter noise, leading to *mismatched decoding*

Analytical Results

Problem Statement

- "Unified" signal model: $\mathbf{y}_j \simeq \mathbf{G}_j \mathbf{H}_{ij} \mathbf{x}_i + \boldsymbol{w}_j$

  where $\boldsymbol{w}_j = \mathbf{G}_j \mathbf{H}_{jj} \boldsymbol{m}_j + \mathbf{G}_j \mathbf{n}_j$ with $\boldsymbol{R}_{\boldsymbol{w}_j} = \frac{\sigma_j^2}{M_j} \mathbf{G}_j \mathbf{H}_{jj} \mathbf{H}_{jj}^H \mathbf{G}_j^H + \mathbf{I}$

  1. Matched decoding uses the true density $p(\mathbf{y}_j \mid \mathbf{x}_i, \mathcal{H}_{ij})$
  2. Mismatched decoding estimates $\boldsymbol{R}_{\boldsymbol{w}_j}$ as $\tilde{\boldsymbol{R}}_{\boldsymbol{w}_j}$
     and uses a postulated density $q(\mathbf{y}_j \mid \mathbf{x}_i, \mathcal{H}_{ij})$

- Generalized mutual information (GMI) is defined as

$$\boxed{I_{\mathsf{gmi}}(\mathbf{y}_j; \mathbf{x}_i) = \sup_{s>0} I_{\mathsf{gmi}}^{(s)}(\mathbf{y}_j; \mathbf{x}_i) = \sup_{s>0} \left(\mathsf{E} \ln q(\mathbf{y}_j \mid \mathbf{x}_i, \mathcal{H}_{ij})^s - \mathsf{E} \ln q^{(s)}(\mathbf{y}_j \mid \mathcal{H}_{ij}) \right)}$$

  where $q^{(s)}(\mathbf{y}_j \mid \mathcal{H}_{ij}) = \mathsf{E}_{\mathbf{x}_i} q(\mathbf{y}_j \mid \mathbf{x}_i, \mathcal{H}_{ij})^s$

- The first term is easy to calculate, yielding

$$I_{\mathsf{gmi}}^{(s)}(\mathbf{y}_j; \mathbf{x}_i) = \left(c - s \, \mathsf{E} \, \mathrm{tr}(\tilde{\boldsymbol{R}}_{\boldsymbol{w}_j}^{-1} \boldsymbol{R}_{\boldsymbol{w}_j}) \right) - \mathsf{E} \ln q^{(s)}(\mathbf{y}_j \mid \mathcal{H}_{ij}),$$

  while the second term needs special tricks as follows

Replica Analysis

- Instead of trying direct calculation, let us take a different route and start by reformulating the difficult term as

$$\mathsf{E} \ln q^{(s)}(\mathbf{y}_j \mid \mathcal{H}_{ij}) = c + \lim_{u \to 0} \frac{\partial}{\partial u} \ln \mathsf{E} Z(\mathbf{y}_j, \mathcal{H}_{ij}; s)^u$$

  where $Z(\mathbf{y}_j, \mathcal{H}_{ij}; s) = \mathsf{E}_{\mathbf{x}_i} \mathrm{e}^{-(\mathbf{y}_j - \mathbf{G}_j \mathbf{H}_{ij} \mathbf{x}_i)^H s \tilde{\boldsymbol{R}}_{\boldsymbol{w}_j}^{-1} (\mathbf{y}_j - \mathbf{G}_j \mathbf{H}_{ij} \mathbf{x}_i)}$
- To circumvent the problem of u being real-valued, the replica trick then postulates

$$Z(\mathbf{x}_0, \boldsymbol{w}_j, \mathcal{H}_{ij}; s)^u = \mathsf{E}_{\{\mathbf{x}_a\}_{a=1}^u} \prod_{a=1}^{u} \mathrm{e}^{-[\boldsymbol{w}_j + \mathbf{G}_j \mathbf{H}_{ij}(\mathbf{x}_0 - \mathbf{x}_a)]^H s \tilde{\boldsymbol{R}}_{\boldsymbol{w}_j}^{-1} [\boldsymbol{w}_j + \mathbf{G}_j \mathbf{H}_{ij}(\mathbf{x}_0 - \mathbf{x}_a)]}$$

  where $\mathbf{x}_0$ and $\{\mathbf{x}_a\}_{a=1}^u$ denote the original and replicated vectors
- If we manage to assess the above expectation as a function of u when matrix dimensions in $\mathcal{H}_{ij}$ grow without bound with fixed ratios, analytically continuing $u \to 0$ recovers the per-stream GMI as

$$\frac{1}{M} I_{\mathsf{gmi}}^{(s)}(\mathbf{y}_j;\, \mathbf{x}_i) = -\frac{s}{M} \mathsf{E}\, \mathrm{tr}(\tilde{\boldsymbol{R}}_{\boldsymbol{w}_j}^{-1} \boldsymbol{R}_{\boldsymbol{w}_j}) - \lim_{M \to \infty} \frac{1}{M} \lim_{u \to 0} \frac{\partial}{\partial u} \ln \mathsf{E} Z(\mathbf{x}_0, \boldsymbol{w}_j, \mathcal{H}_{ij}; s)^u$$

Matched Decoding: Per-stream Achievable Rate

- When $\mathbf{H}_{ij}$ and $\mathbf{H}_{jj}$ are i.i.d. Gaussian with gains $\bar{\gamma}_{ij}$ and $\bar{\gamma}_{jj}$ and the receiver adapts perfectly to residual self-interference:

$$\boxed{\frac{R_{ij}}{M_i} = \ln(1+\eta_{ij}) - \frac{\eta_{ij}}{1+\eta_{ij}} + \frac{1}{\alpha_{ij}}\left[I\left(\alpha_{jj}, \bar{\gamma}_{jj}\sigma_j^2;\, 1+\frac{\bar{\gamma}_{ij}}{1+\eta_{ij}}\right) - I(\alpha_{jj}, \bar{\gamma}_{jj}\sigma_j^2;\, 1)\right]}$$

  for which the fixed-point η_{ij} is found numerically by iterating

$$\eta_{ij} = \frac{\bar{\gamma}_{ij}}{\alpha_{ij}}\left[\frac{1}{1+\frac{\bar{\gamma}_{ij}}{1+\eta_{ij}}} - \frac{\alpha_{ii}}{4\bar{\gamma}_{jj}\sigma_j^2}\mathcal{F}\left(\frac{\bar{\gamma}_{jj}\sigma_j^2}{\alpha_{ii}} \cdot \frac{1}{1+\frac{\bar{\gamma}_{ij}}{1+\eta_{ij}}}, \alpha_{ii}\right)\right]$$

  and the auxiliary functions are given by

$$\mathcal{F}(x,\beta) = \left(\sqrt{x(1+\sqrt{\beta})^2+1} - \sqrt{x(1-\sqrt{\beta})^2+1}\right)^2$$

$$I(\beta,\sigma^2;\, t) = \ln t + \beta\ln\left[1+\frac{\sigma^2}{t\beta} - \frac{1}{4}\mathcal{F}\left(\frac{\sigma^2}{t\beta},\beta\right)\right] + \ln\left[1+\frac{\sigma^2}{t} - \frac{1}{4}\mathcal{F}\left(\frac{\sigma^2}{t\beta},\beta\right)\right] - \frac{t\beta}{4\sigma^2}\mathcal{F}\left(\frac{\sigma^2}{t\beta},\beta\right)$$

- N.B.: This result is for cancellation only

Mismatched Decoding: Per-stream Achievable Rate

- When $\mathbf{H}_{ij}$ and $\mathbf{H}_{jj}$ are i.i.d. Gaussian with gains $\bar{\gamma}_{ij}$ and $\bar{\gamma}_{jj}$ and the receiver postulates imperfectly $\tilde{\boldsymbol{R}}_{\boldsymbol{w}_j} = (1+\bar{\gamma}_{jj}\tilde{\sigma}_j^2)\mathbf{I}_N$:

$$\frac{R_{ij}}{M_i} = -\frac{s(1+\bar{\gamma}_{jj}\sigma_j^2)}{\alpha_{ij}(1+\bar{\gamma}_{jj}\tilde{\sigma}_j^2+s\tilde{E}_{ij})}\cdot\frac{s\tilde{E}_{ij}}{1+\bar{\gamma}_{jj}\tilde{\sigma}_j^2}+\ln\left(1+\frac{s\bar{\gamma}_{ij}}{\alpha_{ij}(1+\bar{\gamma}_{jj}\tilde{\sigma}_j^2+s\tilde{E}_{ij})}\right)+\frac{1}{\alpha_{ij}}\ln\left(1+\frac{s\tilde{E}_{ij}}{1+\bar{\gamma}_{jj}\tilde{\sigma}_j^2}\right)$$

where $\tilde{E}_{ij}$ is directly given as

$$\tilde{E}_{ij} = \frac{s\bar{\gamma}_{ij}-(1+\bar{\gamma}_{jj}\tilde{\sigma}_j^2)}{2s}-\frac{\bar{\gamma}_{ij}}{2\alpha_{ij}}+\sqrt{\frac{(1+\bar{\gamma}_{jj}\tilde{\sigma}_j^2)\bar{\gamma}_{ij}}{s}+\left(\frac{s\bar{\gamma}_{ij}-(1+\bar{\gamma}_{jj}\tilde{\sigma}_j^2)}{2s}-\frac{\bar{\gamma}_{ij}}{2\alpha_{ij}}\right)^2}$$

  - ▷ the case of $\tilde{\sigma}_j^2 = 0$ is illustrated in the numerical examples
  - ▷ asymptotic result at large-system limit: $M_i \to \infty$ and $N_j \to \infty$ while $\frac{M_i}{N_j} \to \alpha_{ij}$ for all $i, j \in \{1, 2\}$ (like in the previous slide)

- Optimization is required for the parameter s though, in order to find more tight lower bounds for the maximum achievable rate

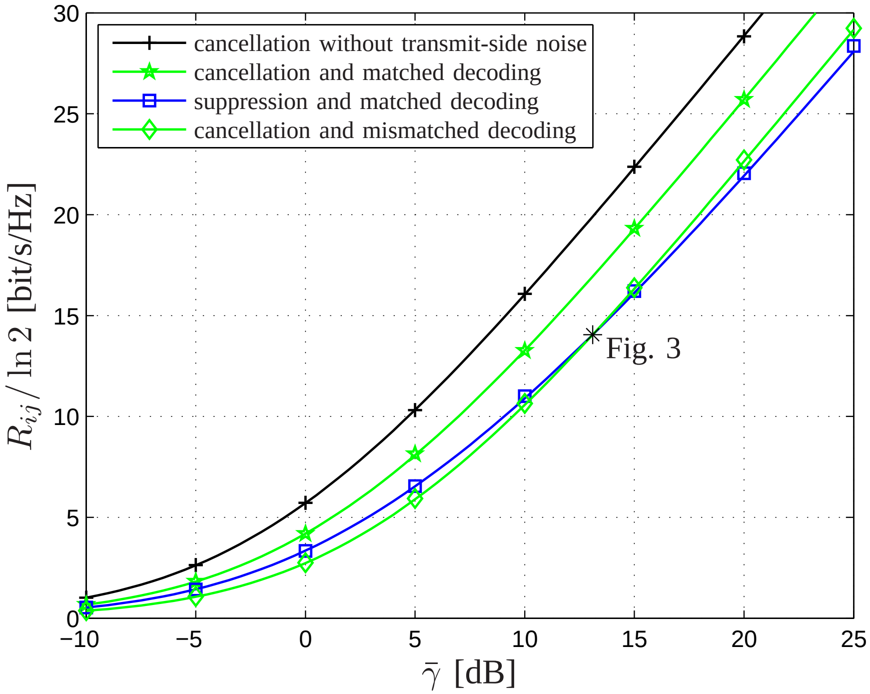
Numerical Examples

Example Setups

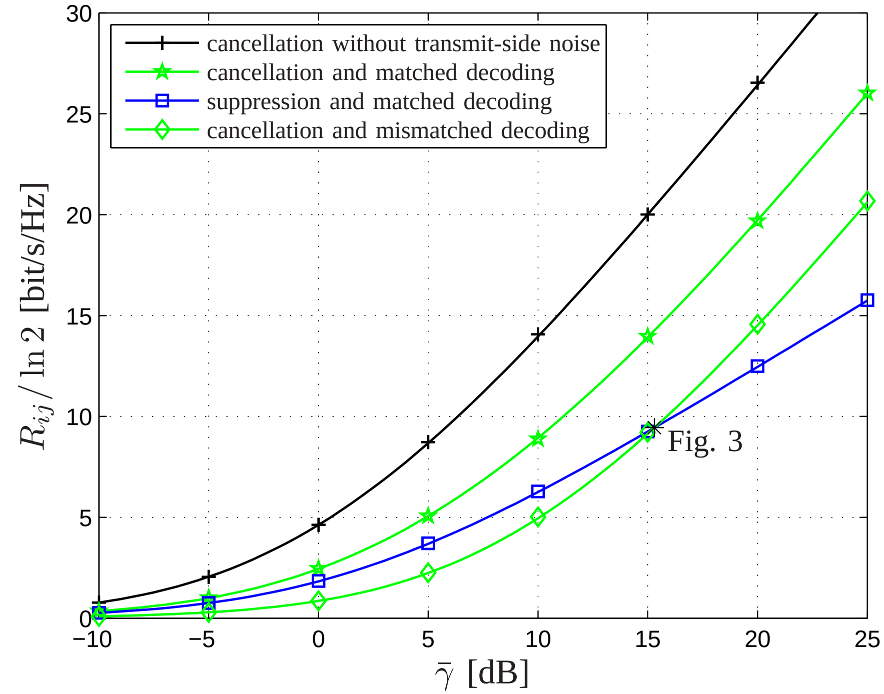
- The numerical results concentrate on symmetric systems where
  - $M = M_1 = M_2$
  - $N = N_1 = N_2$
  - $\bar{\gamma} = \bar{\gamma}_{12} = \bar{\gamma}_{21}$
  - $\bar{\gamma}_{\mathrm{I}} = \bar{\gamma}_{11} = \bar{\gamma}_{22}$
  - $\sigma^2 = \sigma_1^2 = \sigma_2^2$
- There may be transmit/receive antenna imbalance (M/N)
  - Yet M and N grow asymptotically at the large-system limit
- Choice $\sigma^2 = 0.001$ corresponds to transmitter EVM of -30 dB (or equivalently 3.2%) which is a practical but slightly optimistic value
- In summary, there are three key parameters to explore:

$$\boxed{\bar{\gamma}} \quad \boxed{\bar{\gamma}_{\mathrm{I}}} \quad \boxed{M/N}$$

Achievable Rates vs. SNR (Fig. 2)



(a) $M = 4$, $N = 8$, $\bar{\gamma}_{\mathrm{I}} = 33$ dB



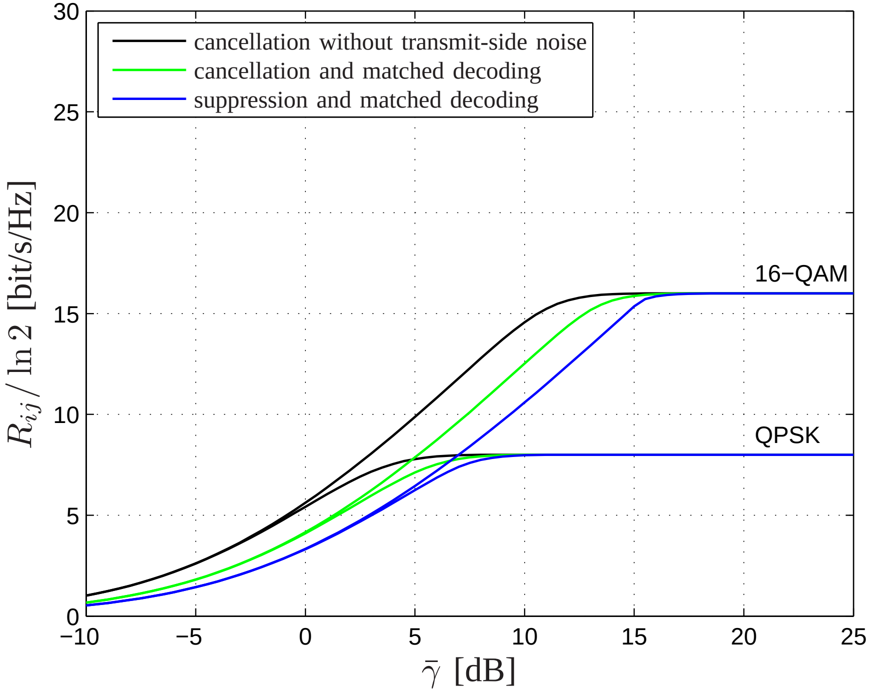
(b) $M = 4$, $N = 6$, $\bar{\gamma}_{\mathrm{I}} = 39$ dB

- Simulations (markers) corroborate analytical results (solid lines)

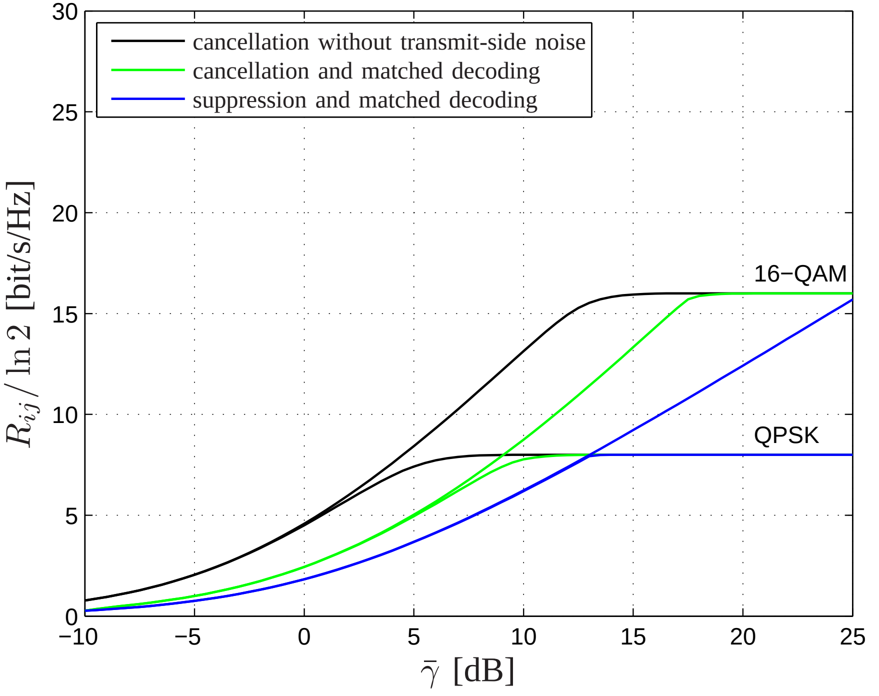
(a) when $M/N \leq 1/2$, suppression reduces receive array gain

(b) when $M/N > 1/2$, suppression reduces multiplexing order

Achievable Rates vs. SNR, Discrete Modulation

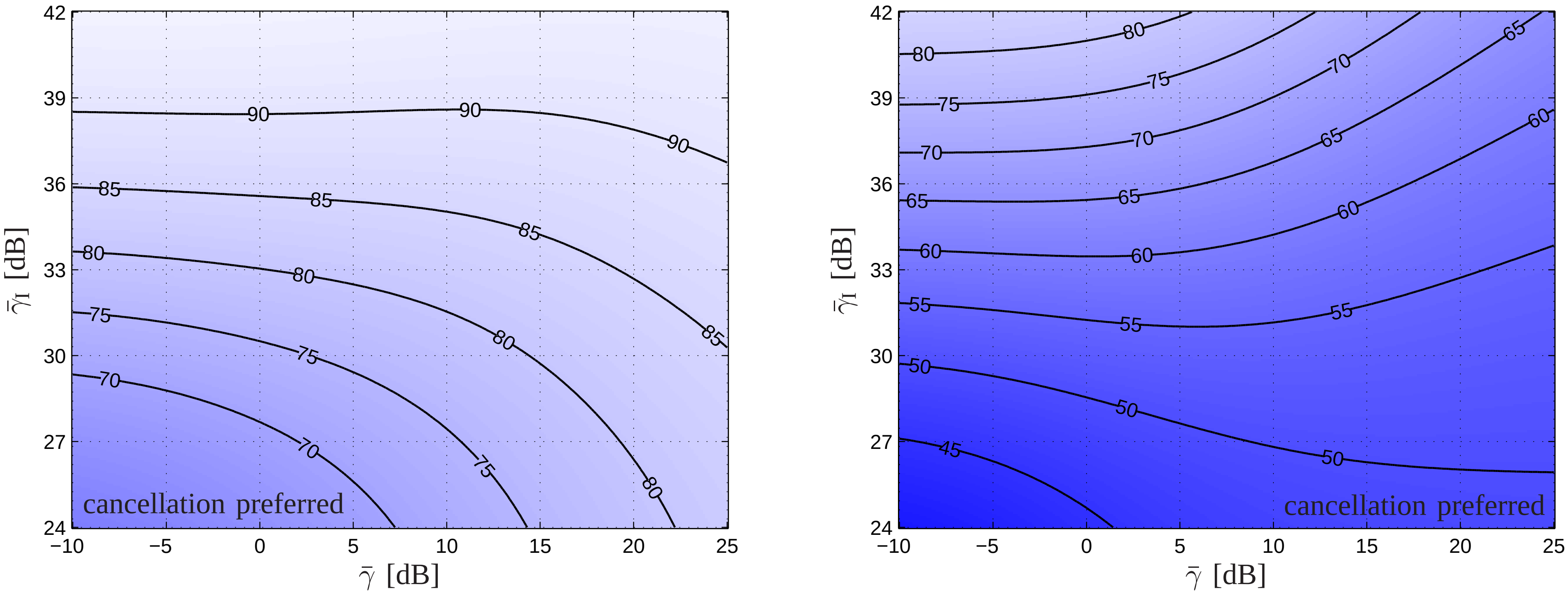


(a) $M = 4$, $N = 8$, $\bar{\gamma}_{\mathrm{I}} = 33$ dB



(b) $M = 4$, $N = 6$, $\bar{\gamma}_{\mathrm{I}} = 39$ dB

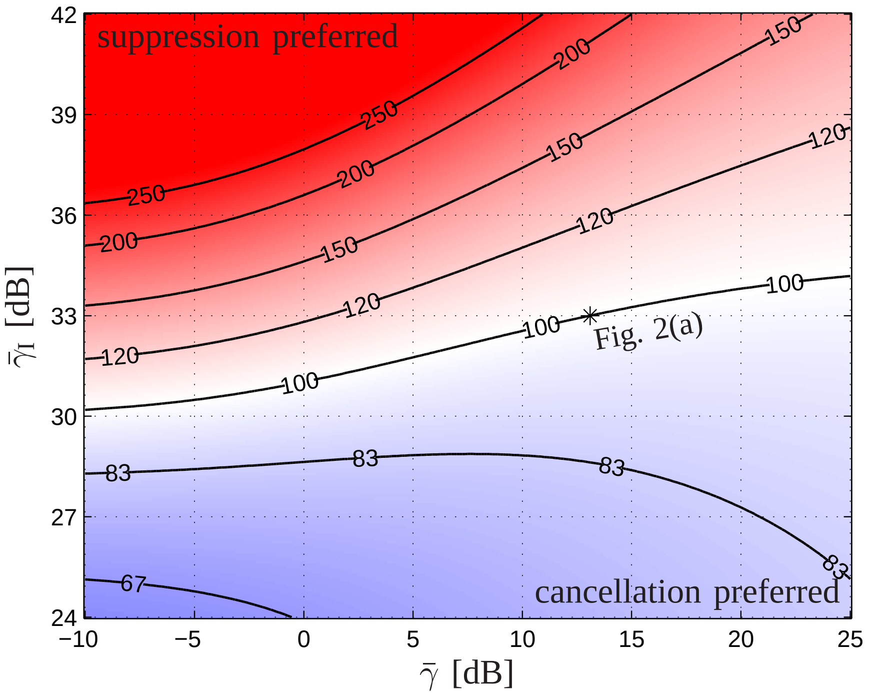
Matched Decoding: Suppression vs. Cancellation [%]



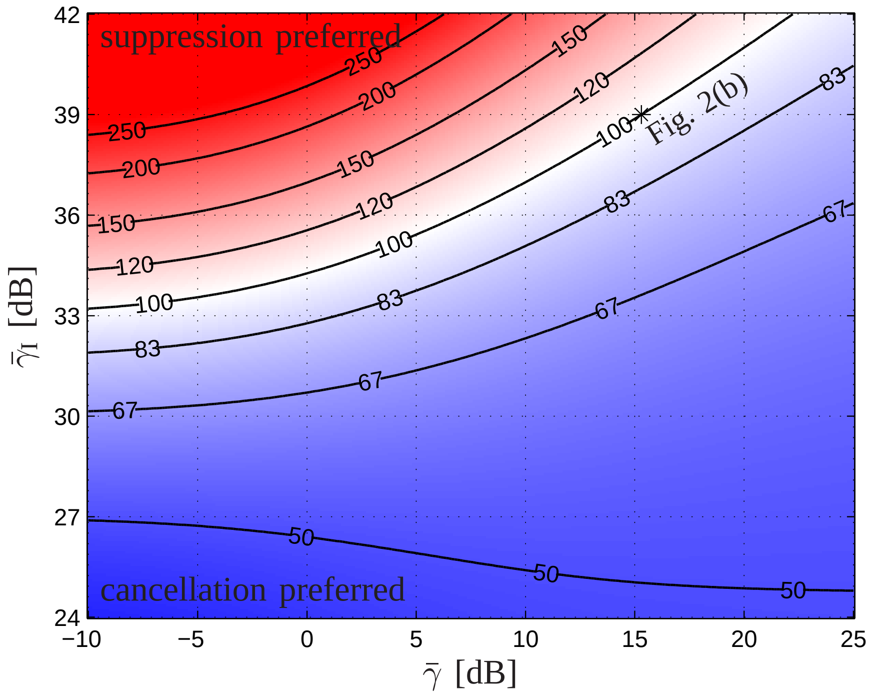
(a) $M/N = 1/2$ (b) $M/N = 2/3$

- Suppression is worse than cancellation if matched decoding is still feasible under residual self-interference, since such receivers already comprise ideal interference and noise control

Mismatched Decoding: Suppression vs. Cancellation [%]



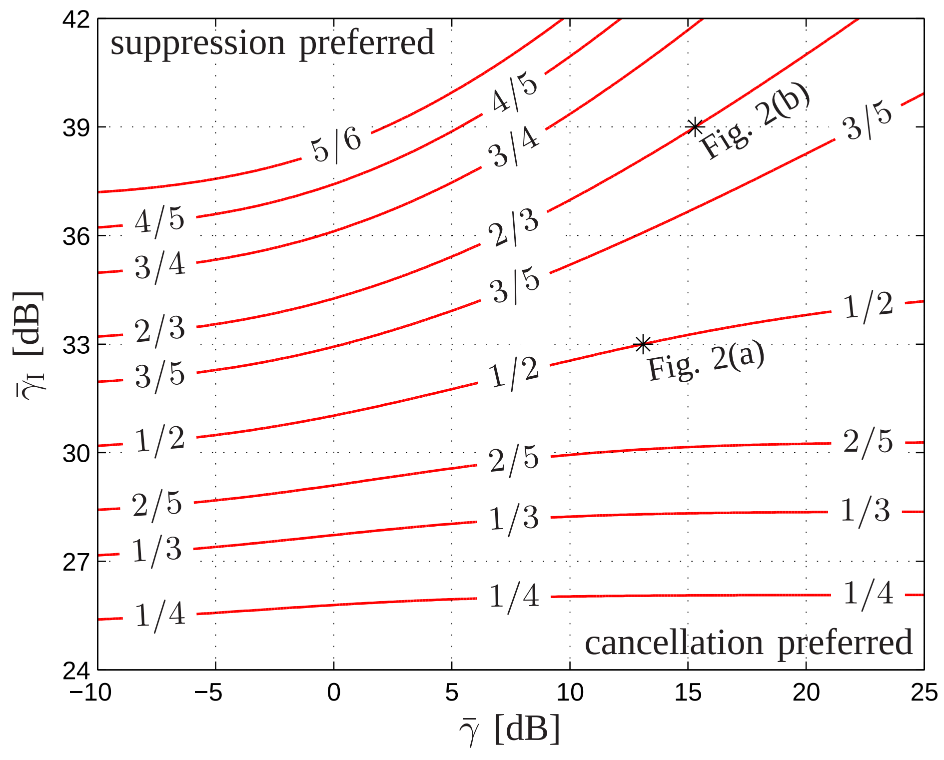
(a) $M/N = 1/2$



(b) $M/N = 2/3$

- Transmitter noise and mismatched decoding cause an intricate interplay between the parameters corresponding to the channel gains of the data and self-interference links and the antenna ratio

Mismatched Decoding: Switching Boundaries



- Suppression becomes preferred in wide SNR range when the number of receive antennas vs. transmit antennas is large
- The level of self-interference is a significant factor at low SNR

Conclusion

Conclusion

- Achievable rates in bidirectional full-duplex link
- Comparison of spatial *suppression* and subtractive *cancellation*
  - for characterizing the cost and benefit of allocating a part of spatial degrees of freedom for self-interference mitigation
  - Trade-off between reduced multiplexing order or array gain and residual self-interference
- *Mismatched decoding* due to *transmitter imperfections*
- Analysis at the large-system limit based on the replica method
  - Monte Carlo simulations with small number of antennas match well with the corresponding asymptotic results

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